Modeling Antarctic Ice with Adaptive Mesh Refinement

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Applied Numerical Algorithms Group

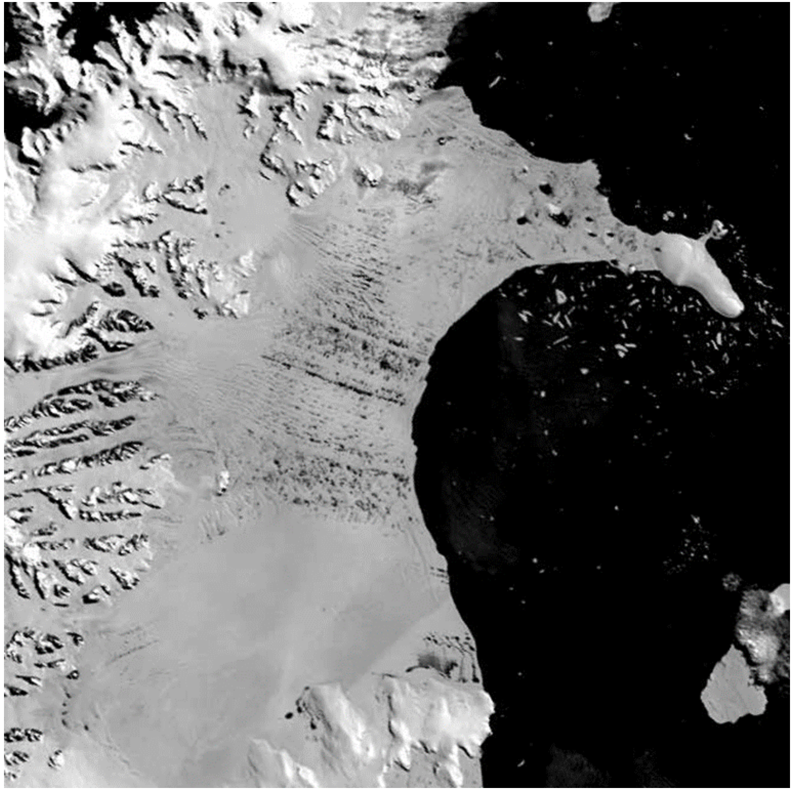
Who I am…

- Grew up outside Philadelphia, HS in Orlando
- Undergrad: Mechanical Engineering at the University of Florida
- Grad school: PhD in Mechanical engineering at UC Berkeley (fluids)
- Came to LBL as a post-doc, never left (22 years ago)
- Currently the group lead for the Applied Numerical Algorithms Group in CRD

- Focus on developing algorithms and software for solving systems of PDEs efficiently and accurately for real applications
- Married, 2 boys (15 and 12)
- Hobbies: cycling, music, travel

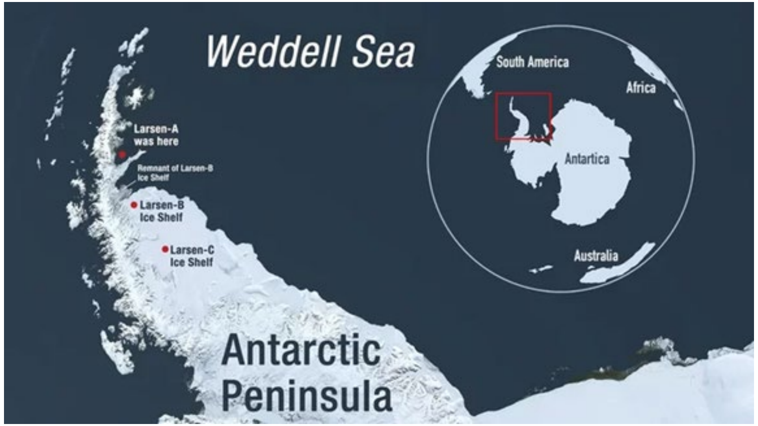
Marine Ice Sheets: Larsen B Breakup (2002)

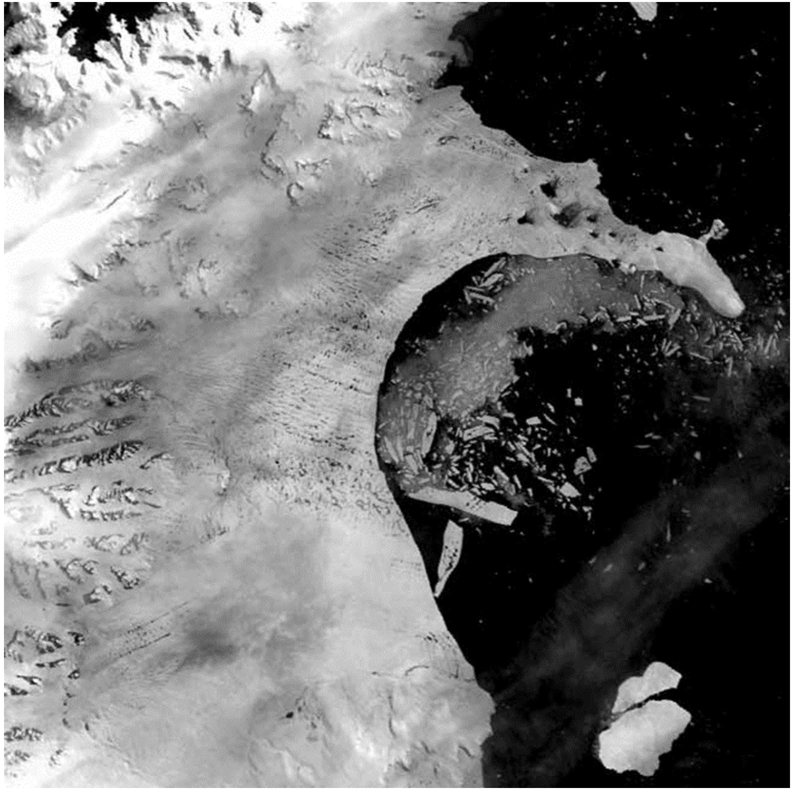
- **January 31, 2002**



Marine Ice Shelves: Larsen B Breakup (2002)

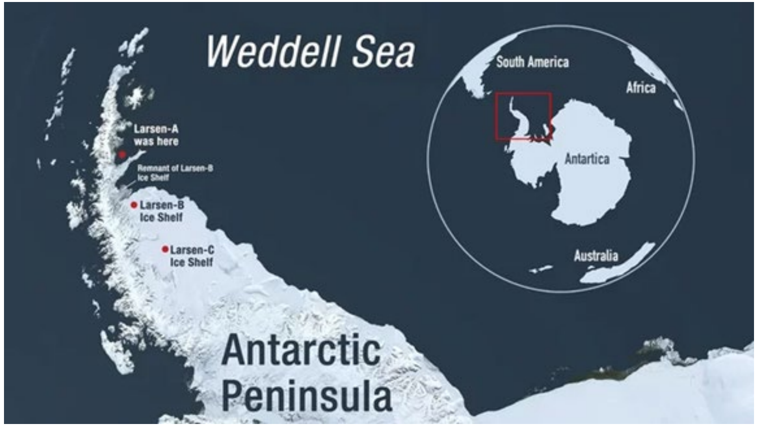
- **February 17, 2002**

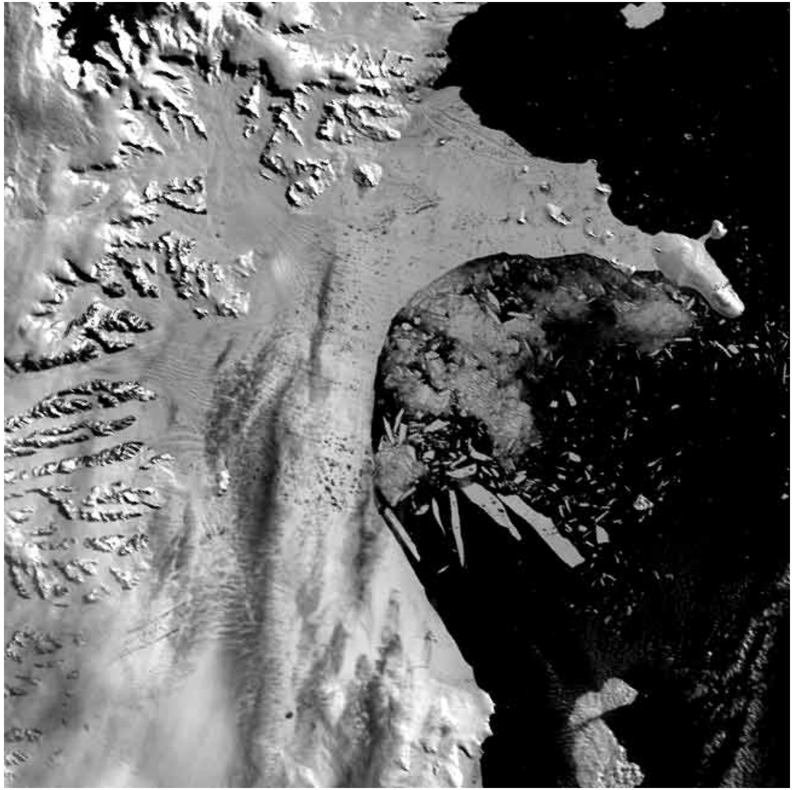




Marine Ice Shelves: Larsen B Breakup (2002)

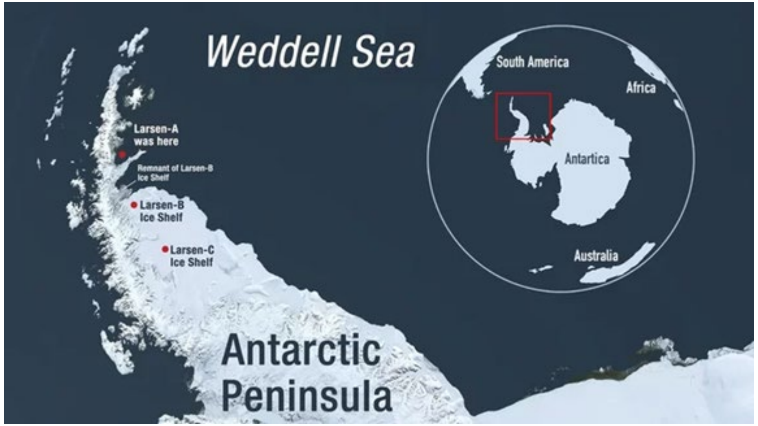
- **February 23, 2002**

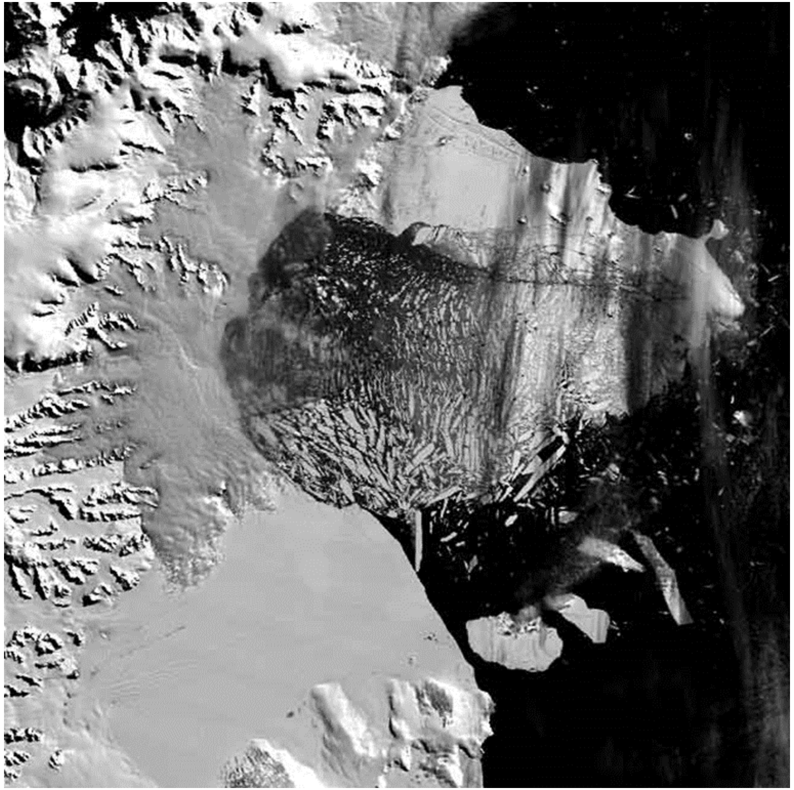




Marine Ice Shelves: Larsen B Breakup (2002)

- **March 5, 2002**





Aftermath…

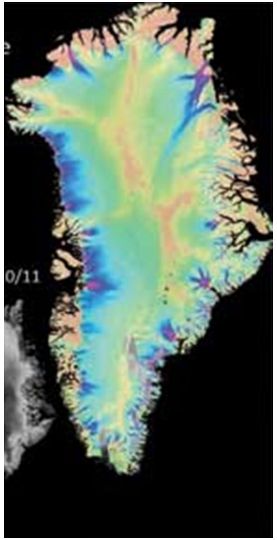
- **3,250 square kilometers (1,250 square miles)**
- **Breakup took about 1 month**
- **Likely due to exceptionally warm summer**
  - Melt pools on surface –- surface melting -> hydrofracture
  - Warm ocean temperatures in the Weddell Sea

- **Results: Larsen A and B glaciers**
  - **abrupt acceleration, about 300% on average**
  - mass loss went from **2–4 gigatonnes per year** in 1996 and 2000 (gigatonne = one billion metric tonnes), to between **22 - 40 gigatonnes per year** in 2006.
  - Not the last! (Wilkins, 2008-2009)

Why do we care? Currently two ice sheets…

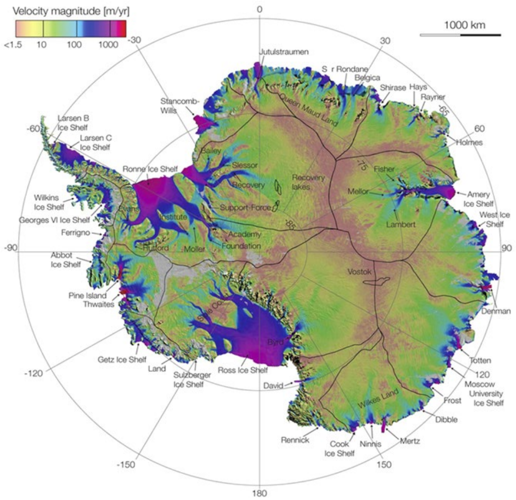
Greenland Ice Sheet

5-7 m Sea Level Equivalent (SLE)



Antarctic Ice Sheet

57 m SLE (4-5m in marine-grounded parts of West Antarctica)



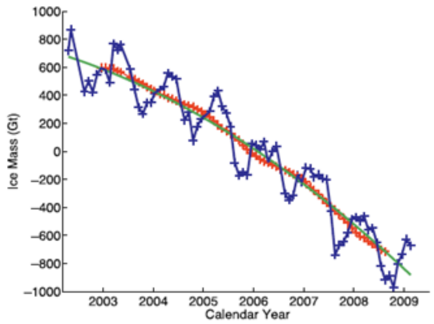
BERKELEY LAB

Why do we care?

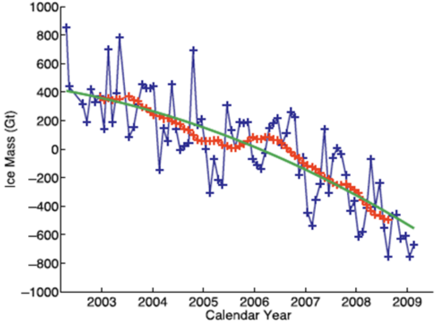
Global Sea Level Budget:

- **Ocean thermal expansion**: ~1 mm/yr
- **Glaciers and ice caps**: ~1 mm/yr
- **Ice sheets**: ~1 mm/yr
  - Greenland 0.6 mm/yr
  - Antarctica 0.4 mm/yr
- **Terrestrial storage**: ~0 mm/yr
  - Dam retention -0.3 mm/yr
  - Groundwater depletion 0.3 mm/yr

The ice sheet contribution has roughly **doubled** since 2000 and will likely continue to increase.



Greenland ice mass loss



Antarctic ice mass loss
(Velicogna 2009)

State of the art, 2007

- **IPCC AR4: called out existing ice sheet modeling state of the art as inadequate**

- **DOE ASCR response:**
  - Call for next-generation ice sheet model development
  - **ISICLES:** 6 (small) funded projects (O(1-2 FTE) each)
  - included LBL-led AMR effort: Berkeley-ISICLES (**BISICLES**), engaged with BER-funded climate scientists & glaciologists at LANL…
  - Synergy with similar Bristol (UK)-led effort (1 postdoc)

How Ice sheets work…

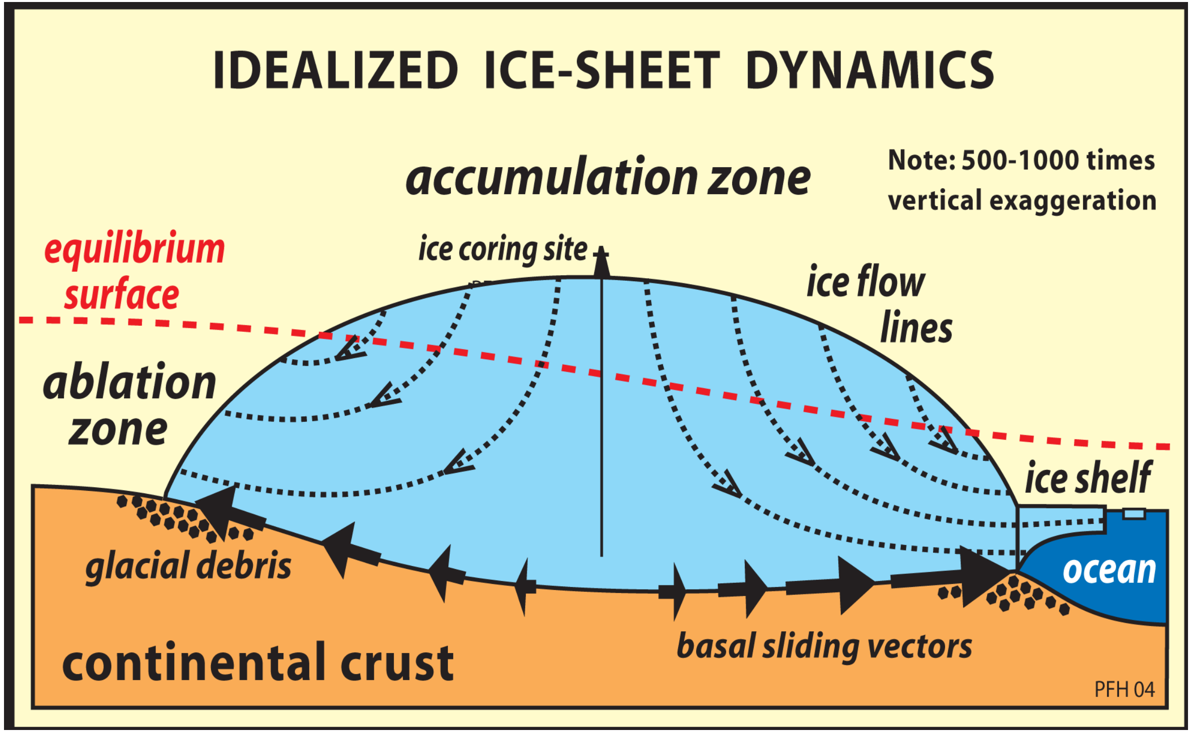
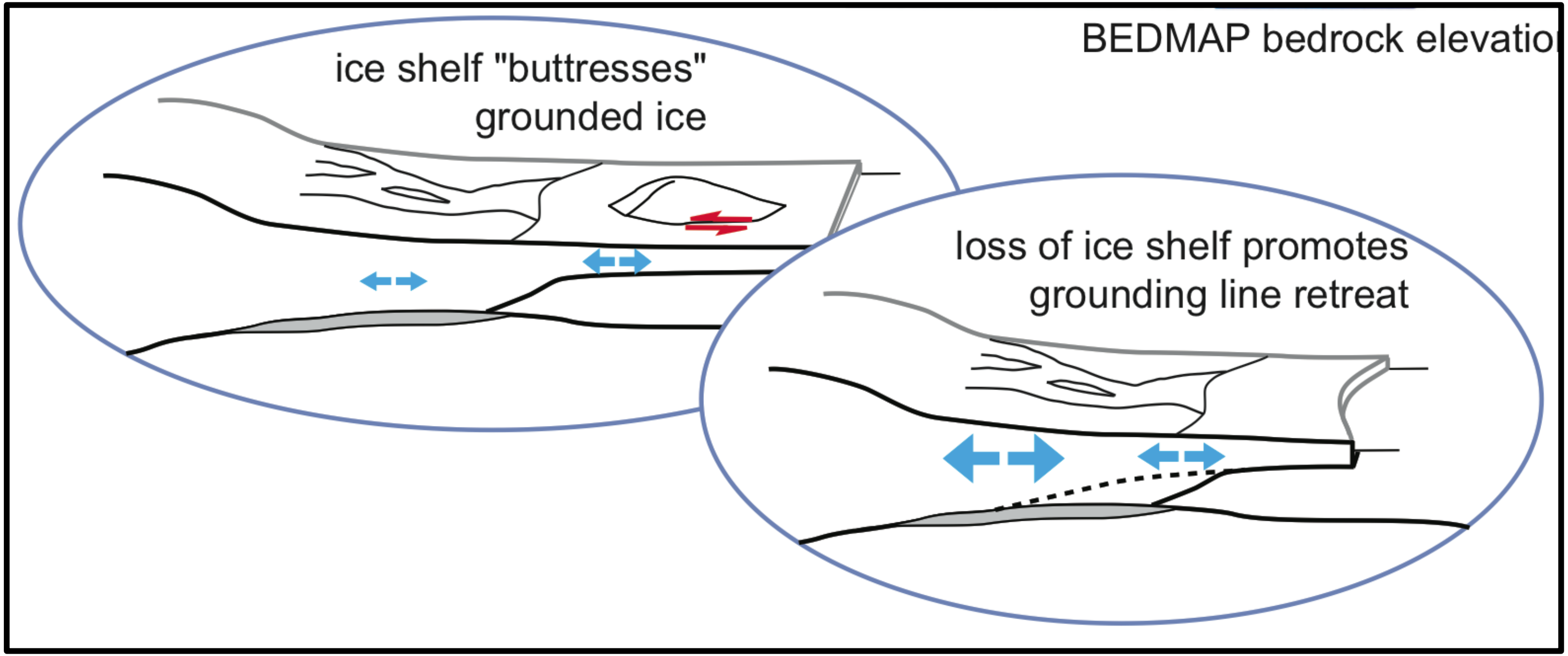
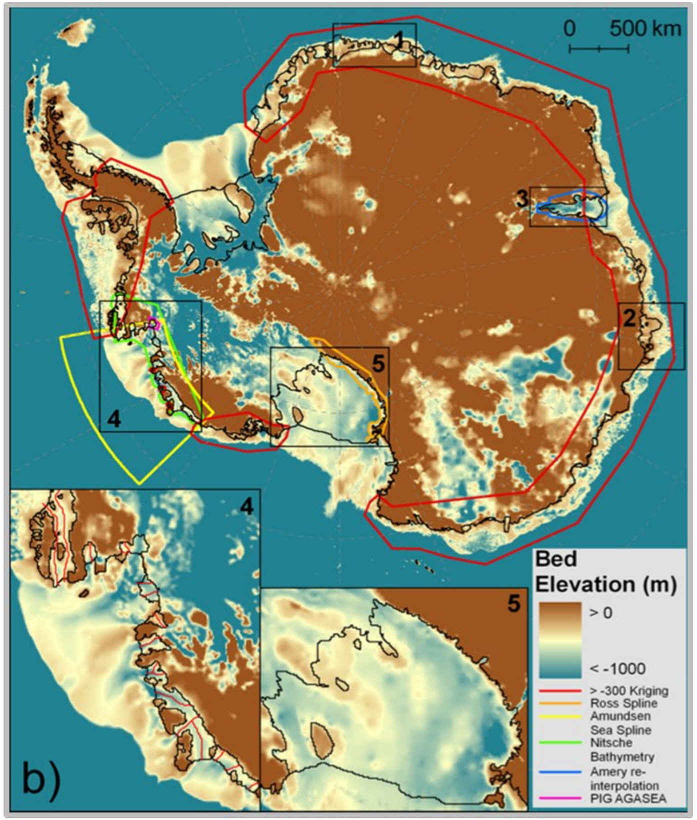


Image: http://www.snowballearth.org

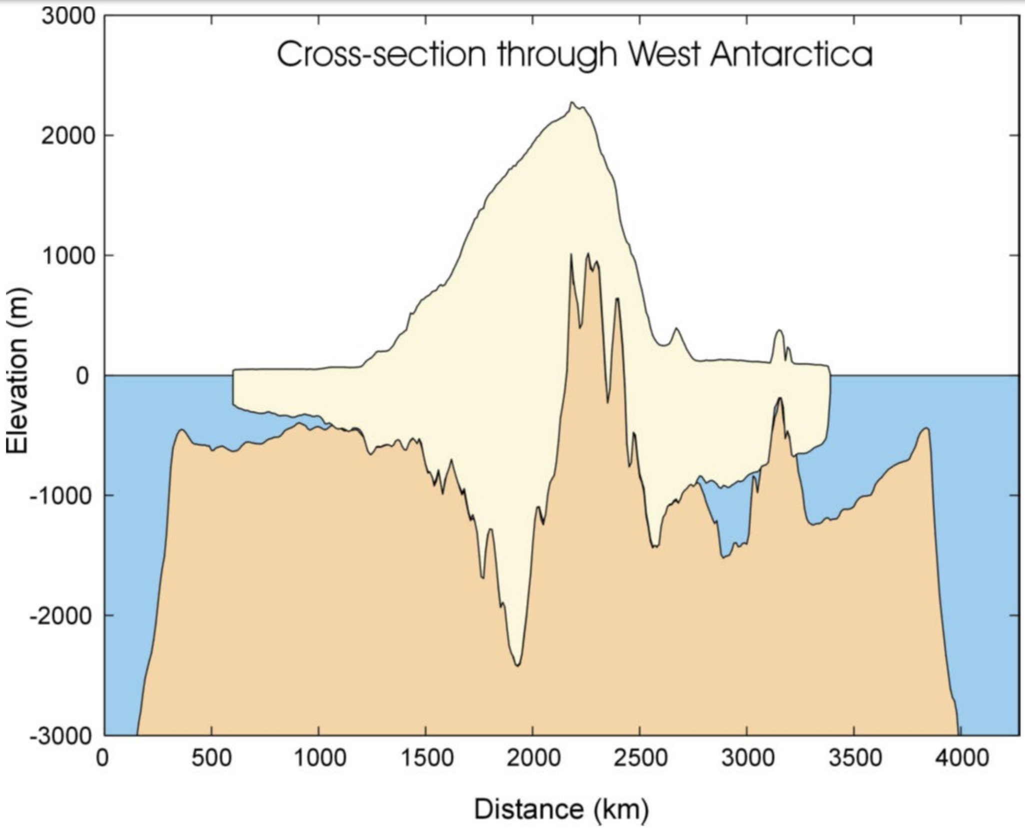
Antarctic Marine Ice Sheet Instability



Antarctic Marine Ice Sheet Instability



LeBrocq et al., 2010


Cross-section through West Antarctica
Elevation (m)
Distance (km)
3000
2000
1000
0
-1000
-2000
-3000
0
500
1000
1500
2000
2500
3000
3500
4000

What does an ice sheet model look like?

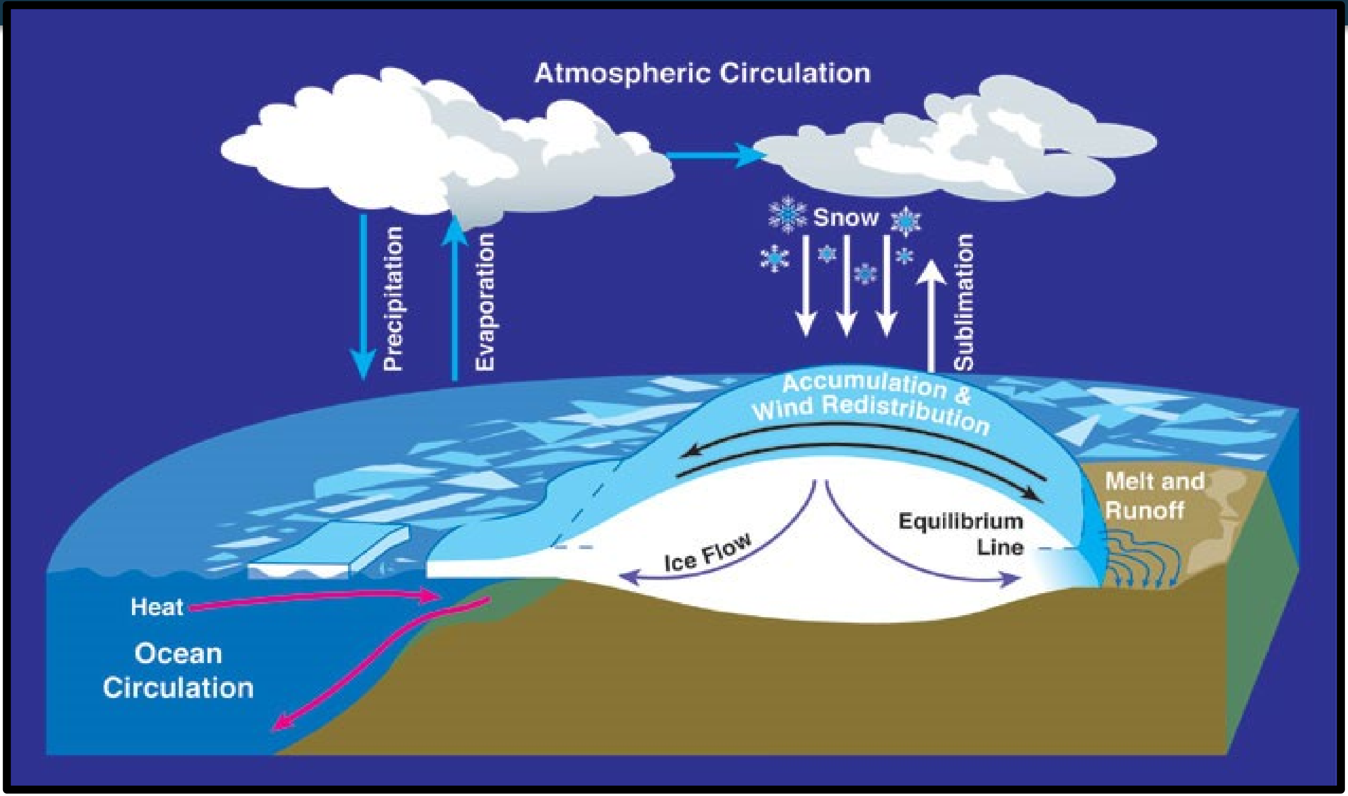


Image source: http://www.nasa.gov/images/content/53743main_atmos_circ.jpg

$$\frac{\partial H}{\partial t} + \nabla \cdot (\vec{u}H) = B - m$$

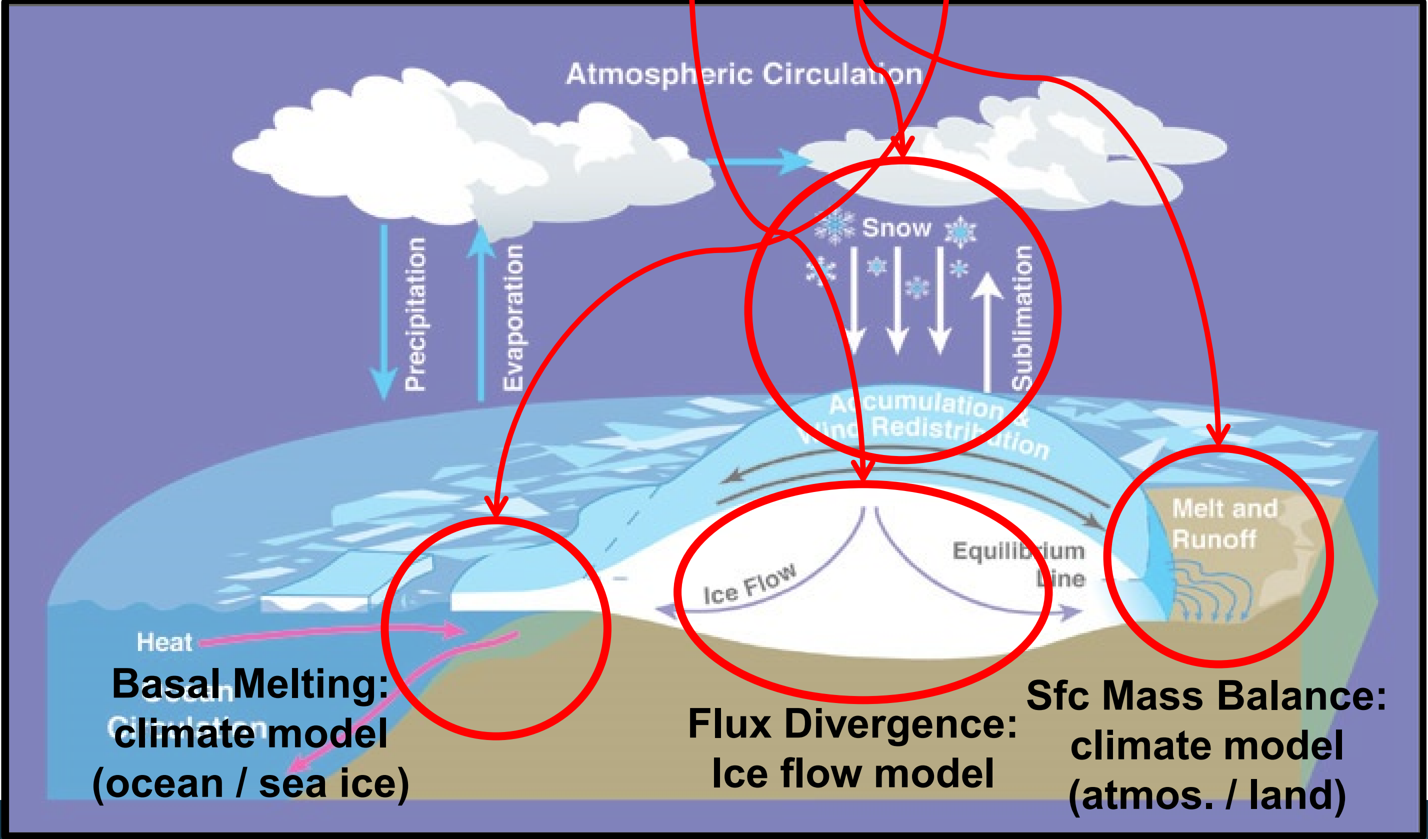


Image source: http://www.nasa.gov/images/content/53743main_atmos_circ.jpg

Models and Approximations

Physics: Non-Newtonian viscous flow: $\mu(\dot{\epsilon}^2$,T) = A(T)$(\dot{\epsilon}^2)^{\frac{(1-n)}{2}}$

- **Full-Stokes**
  - Best fidelity to ice sheet dynamics
  - Computationally expensive (full 3D coupled nonlinear elliptic equations)
- **Approximate Stokes**
  - Use scaling arguments to produce simpler set of equations
  - Common expansion is in ratio of vertical to horizontal length scales $\left(\varepsilon = \frac{[h]}{[l]}\right)$
  - E.g. Blatter-Pattyn (most common "higher-order" model), accurate to O(ε^2)
  - Still 3D, but solve simplified elliptic system (e.g. 2 coupled equations)
- **Depth-integrated**
  - "Shallow Ice" and "Shallow-Shelf" approximations (accurate to O(ε))
  - Special case of approximate Stokes with 2D equation set
  - Easiest to work with computationally, generally less accurate

"L1L2" Model (Schoof and Hindmarsh, 2010)

- **Uses asymptotic structure of full Stokes system to construct a higher-order approximation**
  - Expansion in ε= $\frac{[H]}{[L]}$ and λ= $\frac{[\tau_{shear}]}{[\tau_{normal}]}$ (ratio of shear & normal stresses)
    - Large λ: shear-dominated flow
    - Small λ: sliding-dominated flow
  - Computing velocity to $O(\varepsilon^2)$ only requires τ to $O(\varepsilon)$
- **Computationally much less expensive -- enables fully 2D vertically integrated discretizations. (can reconstruct 3d)**
  - Recovers proper fast- and slow-sliding limits:
    - SIA $(1 \ll \lambda \leq \varepsilon^{-1/n})$ -- accurate to $O(\varepsilon^2 \lambda^{n-2})$
    - SSA $(\varepsilon \leq \lambda \leq 1)$ – accurate to $O(\varepsilon^2)$

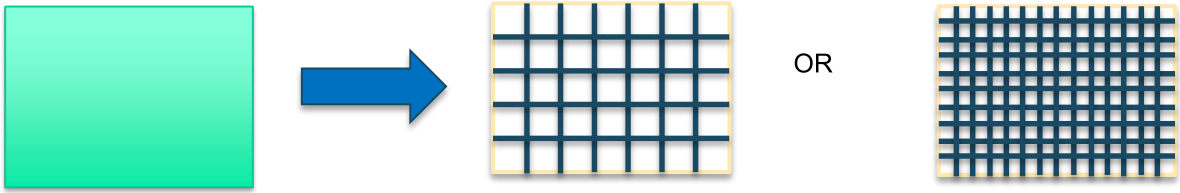
Discrete math for simulations

- Computers are really good at simple arithmetic (+,-, ×, ÷)
- The equations in physics are usually continuous and complicated. (heat conduction:

$$\frac{\partial T}{\partial t} = k\,\nabla^2 T = k\,\frac{\partial^2 T}{\partial x^2} + k\,\frac{\partial^2 T}{\partial y^2}$$

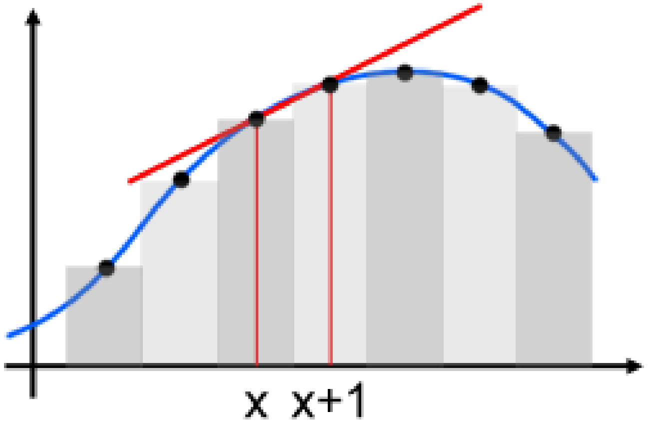
where *T* = temperature, *t* = time, *k* is the "conductivity" (a physical property)

- So, we break up the world into small pieces (*cells*) by laying down a *computational mesh* in the *domain* and then approximating our continuous calculus equations by a bunch of simpler arithmetic ("discrete") ones for each cell. (cell spacing = "Δx")



Discrete Math for Simulations (cont)

Example -- approximating derivatives:

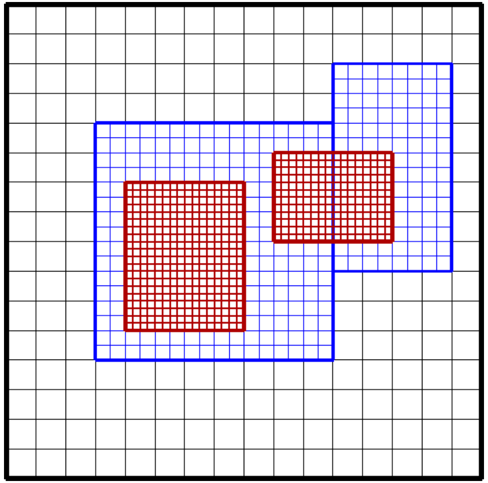


$$\frac{df}{dx} \approx \frac{\Delta f}{\Delta x}$$

$$= \frac{(f_{i+1} - f_i)}{(x_{i+1} - x_i)} \text{ or } \frac{(f_{i+1} - f_{i-1})}{2\Delta x} \text{ or } \frac{(f_i - f_{i-1})}{\Delta x}$$

Adaptive Mesh Refinement

- The finer the mesh, the more accurate your simulation is (like more pixels in a digital camera).
- Finer mesh = more "unknowns" to solve for.
- More unknowns = more expensive to solve.
  (in 3d, $\Delta x \rightarrow \frac{\Delta x}{2}$ means 16x cost!)
- Often, need fine mesh (higher resolution) in some places, but not in others.
- *Adaptive Mesh Refinement* (AMR): Local refinement of computational mesh to improve accuracy only where important.

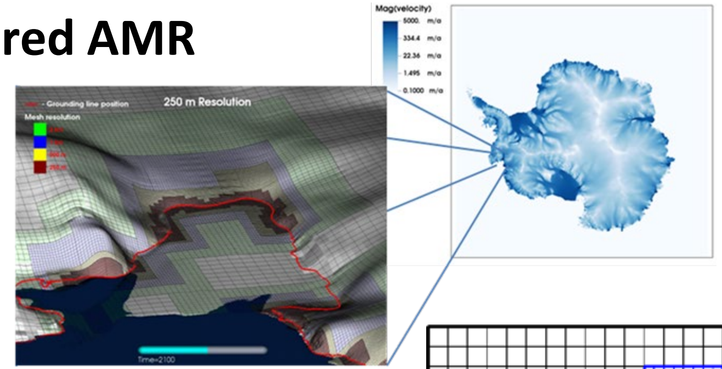


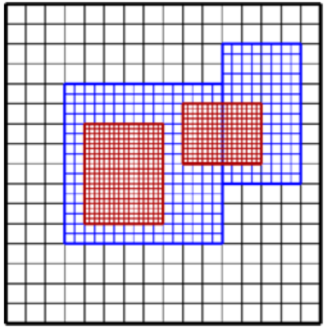
(Figure courtesy of Ann Almgren)

- AMR lets you *dynamically* focus computational effort where you need it

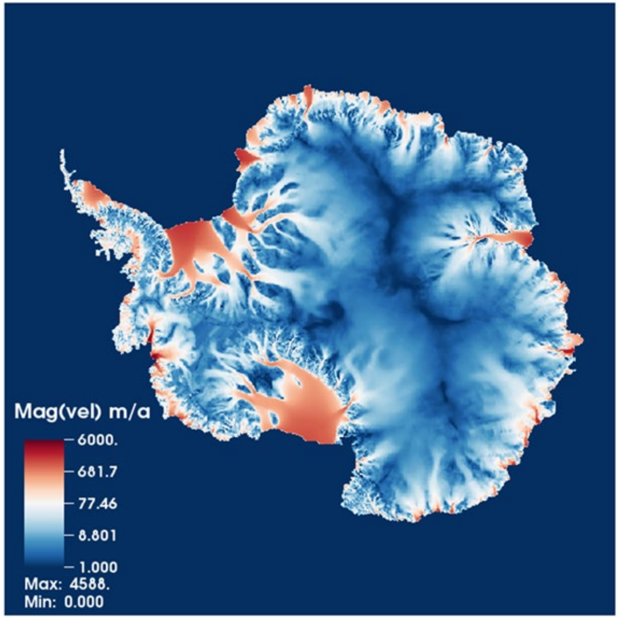
BISICLES Ice Sheet Model

- **Scalable adaptive mesh refinement (AMR) ice sheet model**
  - Dynamic local refinement of mesh to improve accuracy
- **Chombo AMR framework for block-structured AMR**
  - Support for AMR discretizations
  - Scalable solvers
  - Developed at LBNL
  - DOE ASCR supported (FASTMath)
- **Collaboration with Bristol (U.K.) and LANL**
- **Variant of “L1L2” model (Schoof and Hindmarsh, 2010)**
- **Now in second-round of SciDAC funding (PISCEES, ProSPect)**
- **Users in Berkeley, Bristol, Beijing, Brussels, and Berlin…**





Why is this useful? (another BISICLE for another fish?)

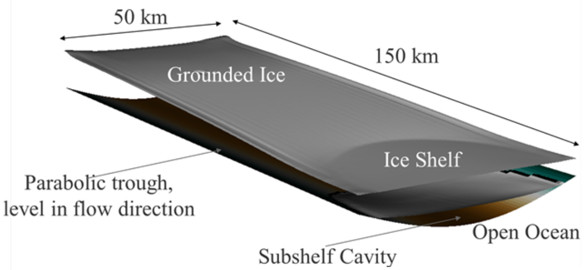


- Ice sheets -- Localized regions where high resolution needed to accurately resolve ice-sheet dynamics (500 m or better at grounding lines)
- Antarctica is really big – too big to resolve at that level of resolution.
- Large regions where such fine resolution is unnecessary (e.g. East Antarctica)

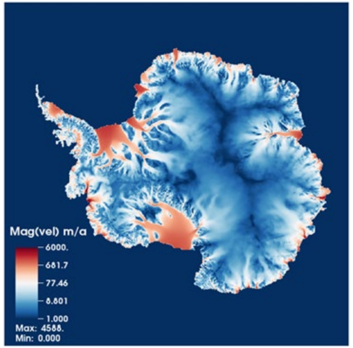
- Well-suited for adaptive mesh refinement (AMR)
- Problems still large: need good parallel efficiency
- Dominated by nonlinear coupled elliptic system for ice velocity solve: good linear and nonlinear solvers

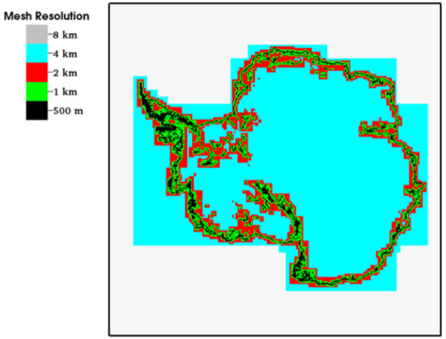
Target Problems

- **Idealized Ice-Ocean interaction test problems**
  - Simple/small geometries designed to understand GL dynamics and ice-ocean interactions
  - MISMIP3D, MISMIP+, MISOMIP



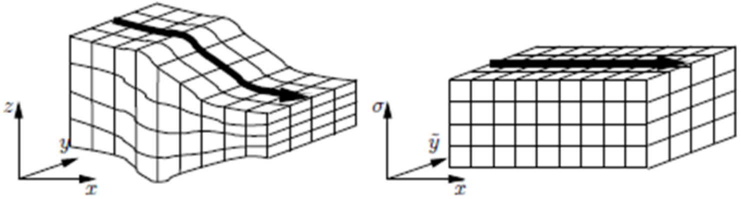
- **Realistic full-scale**
  - Fully-resolved (500m) full-continent
  - Antarctica





Discretizations

- **Baseline model:**
  - Logically-rectangular grid, obtained from a time-dependent uniform mapping.



  - 2D equation for ice thickness *H:*

$$\frac{\partial H}{\partial t} = b - \nabla \cdot (H\overrightarrow{u})$$

  - Vertically-integrated momentum balance results in 2D **nonlinear** viscous tensor solve (viscosity a function of velocity) for velocity $\overrightarrow{u_b}$ at the base of the ice:

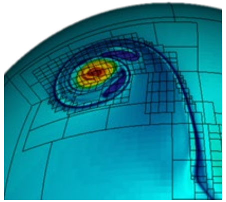
$$\beta^2 \overrightarrow{u_b} + \nabla \cdot \left[\mu(\dot{\varepsilon}^2)\left(\vec{\nabla} + \vec{\nabla}^T\right)\overrightarrow{u_b} - 2\mu\,(\nabla \cdot \overrightarrow{u_b}) \right] = -\frac{g}{\rho} H\,\vec{\nabla} s$$

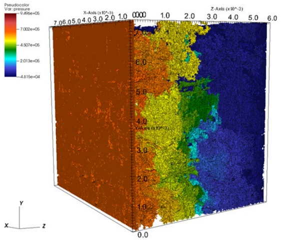
β^2 = friction coefficient, $\dot{\varepsilon}$= strain rate invariant of ice velocity, g = gravity, ρ = ice density, H = ice thickness, $\vec{\nabla} s$ = horizontal gradient of upper surface

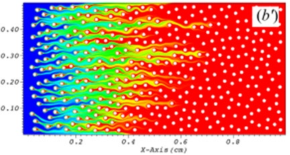
  - Enthalpy formulation for energy

Chombo – Scalable Adaptive Mesh Refinement (AMR)

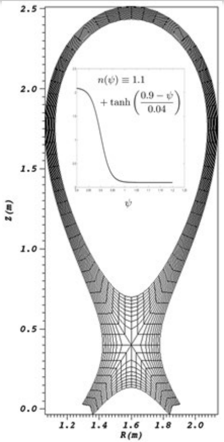
Scalable adaptive mesh refinement (AMR) framework.
Enables implementing scalable AMR applications with support for complex geometries.

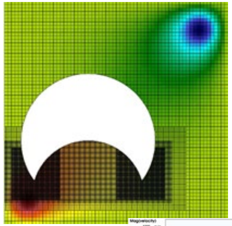




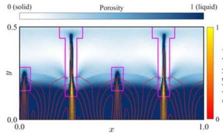


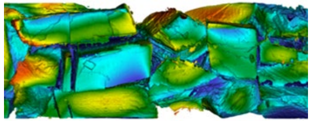
- **Adaptive Mesh Refinement (AMR)**
  - Block structured AMR dynamically focuses computational effort where needed to improve solution accuracy
  - Designed as a developers' toolbox for implementing scalable AMR applications
  - Implemented in C++/Fortran
  - Solvers for hyperbolic, parabolic, and elliptic systems of PDEs
- **Complex Geometries**
  - Embedded-boundary (EB) methods use a cut-cell approach to embed complex geometries in a regular Cartesian mesh
  - EB mesh generation is extremely efficient
  - Structured EB meshes make high performance easier to attain
- **Higher-order finite-volume**
  - Higher (4th)-order schemes reduce memory footprint and improve arithmetic intensity
  - Good fit for emerging architectures
  - Both EB and mapped-multiblock approaches to complex geometry

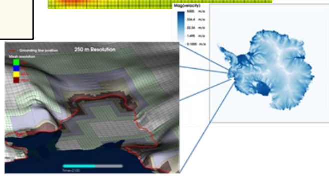




http://Chombo.lbl.gov

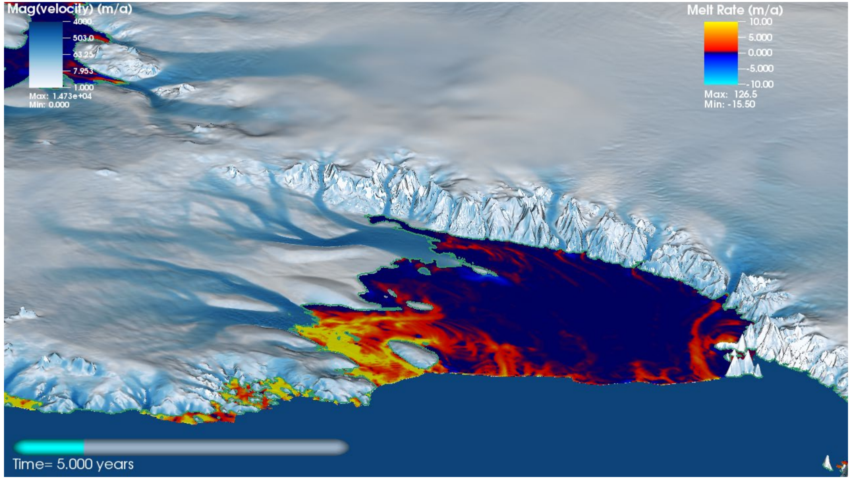


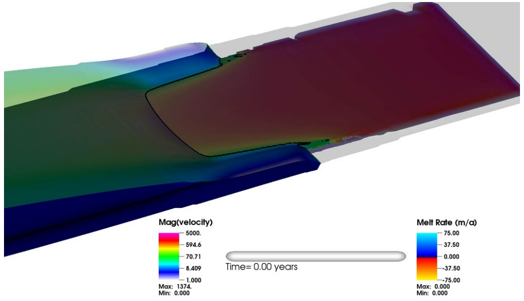




So what can we do with an AMR ice sheet model?

- Couple with ocean & earth system models…





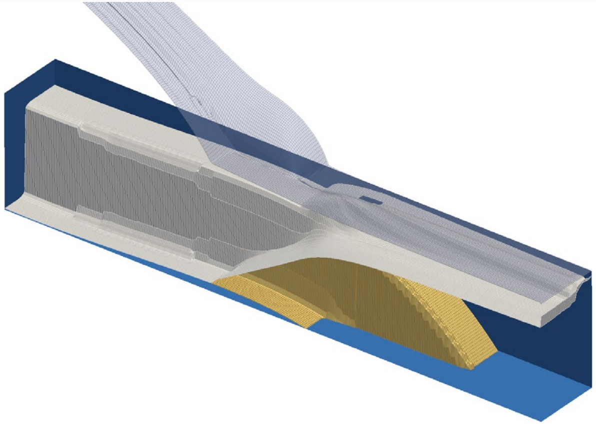
Ice-ocean coupled models:

- **Ocean Circulation Model: POP2x**

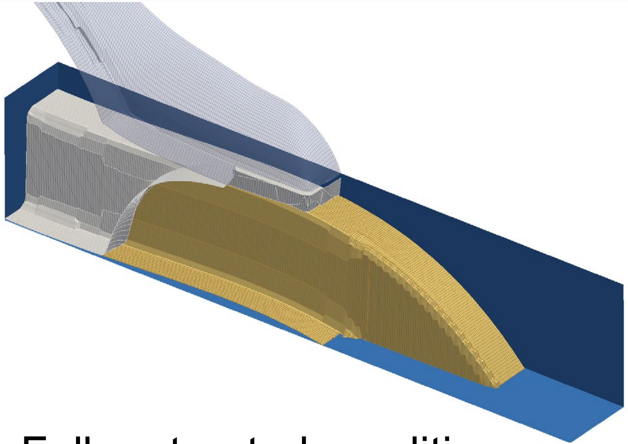
- **Ice Sheet: BISICLES**

- **POP + BISICLES = *POPSICLES***

MISOMIP (Asay-Davis et al (2015))



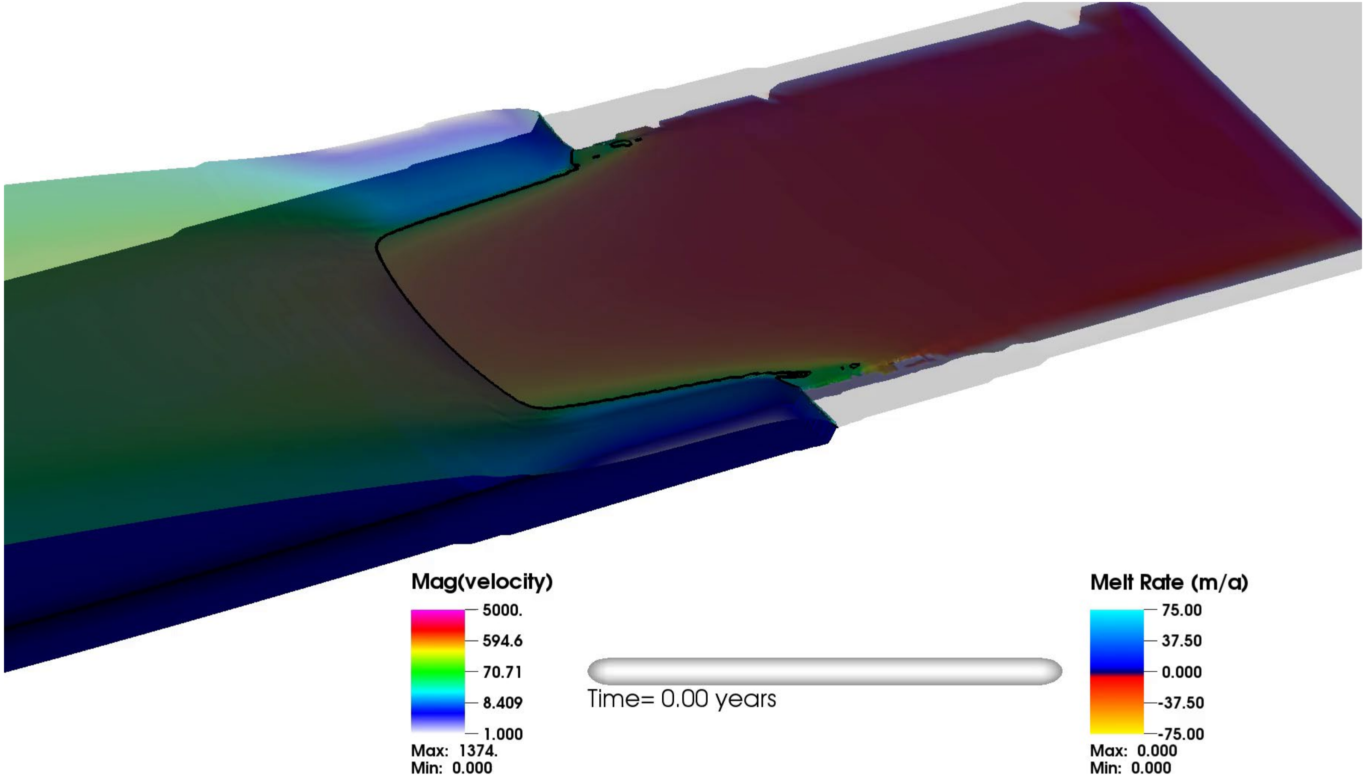
Steady-state initial condition



Fully-retreated condition

- Marine Ice Sheet-Ocean Model Intercomparison Project
- Ice Sheet coupled to Ocean Model through melt rates
- Driven by far-field forcing –
  - 0 < t < 100 years: Warm Phase (1 C)
  - 100 < t < 200 years: Cold Phase (-1.9 C)

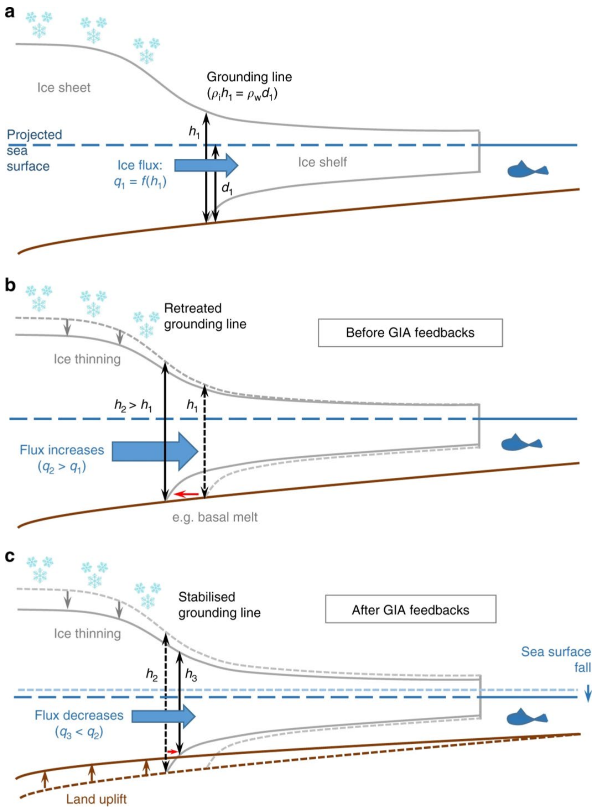
“MISOMIP” test problem



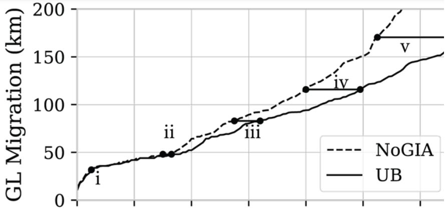
“MISOMIP” test problem

Solid-earth response coupling

- Loading and unloading due to changes in ice sheets causes a response in the solid earth under the ice
  - Glacial Isostatic Adjustment (GIA)
  - Unloading -> “rebound”
- Rebound could potentially help stabilize the ice sheet
- Depends on:
  - Mantle properties
  - Rate of ice sheet change

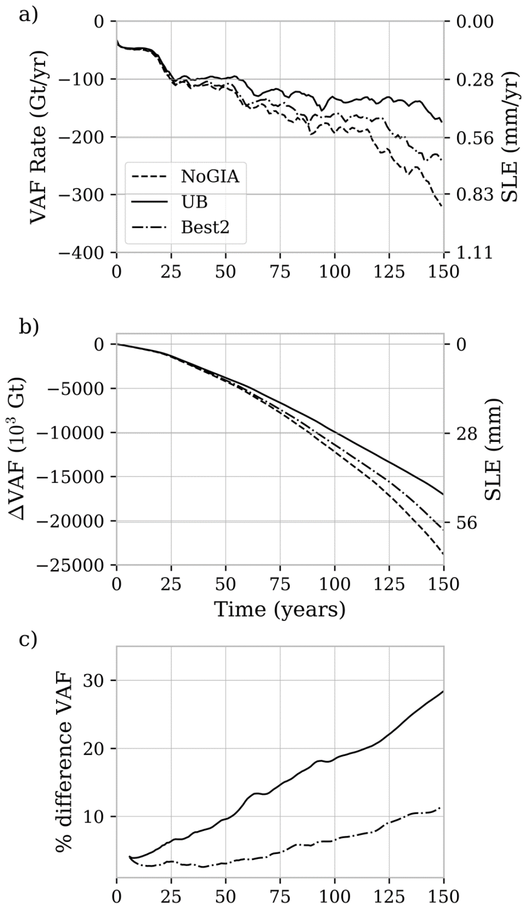


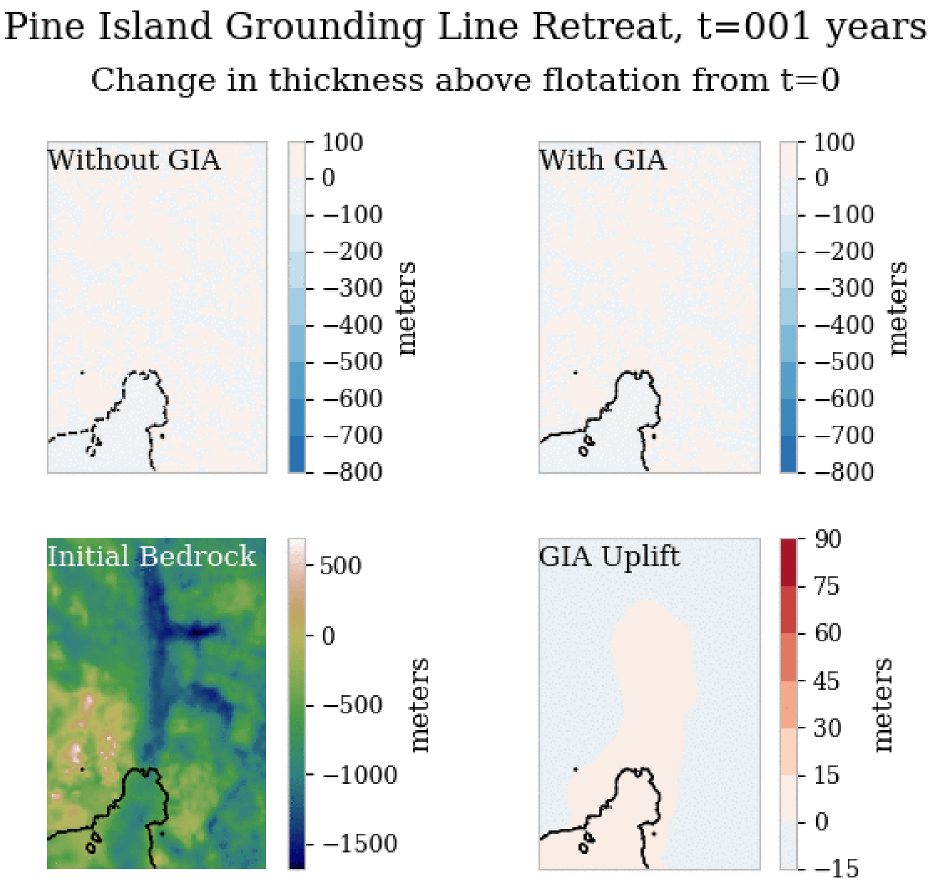
Solid earth response coupling (cont)



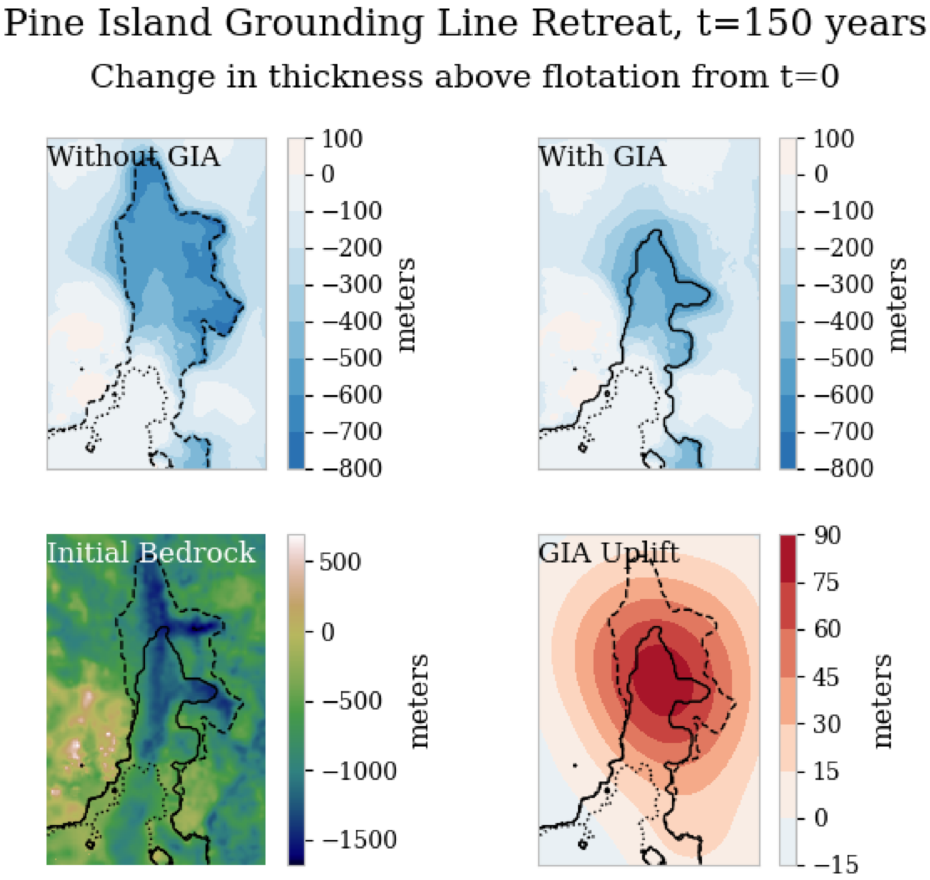
- Recent work: coupled BISICLES to a solid earth model to study impacts
- Some evidence that mantle under WAIS is hotter and weaker than elsewhere
- Result: including coupling with GIA can slow retreat and reduce contribution to sea level rise.
- **Timescale is crucial –** requires that retreat occur at similar timescale as GIA response
  - If retreat is too fast, GIA response is too slow to have an impact
  - Essential that we get the timescales of ice sheet response correct!

(Kachuck, Martin, Bassis, and Price, GRL, 2020)



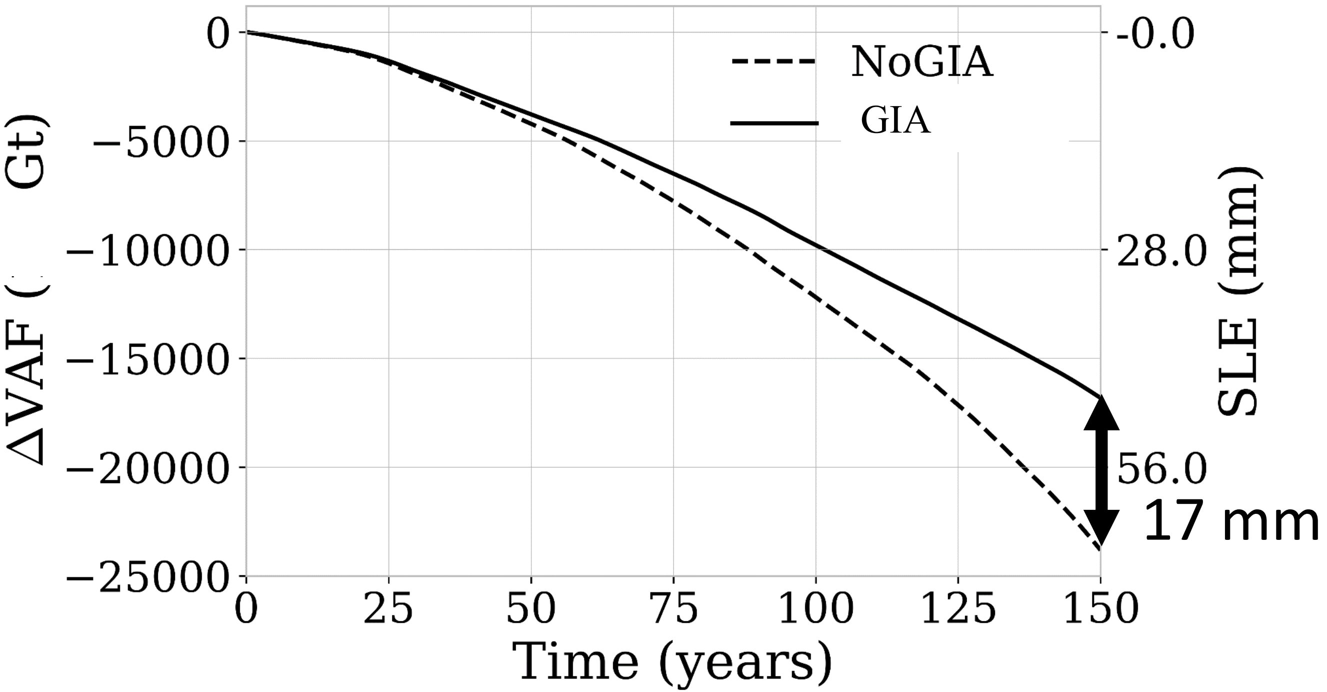


Kachuck, et al. (GRL, 2020)



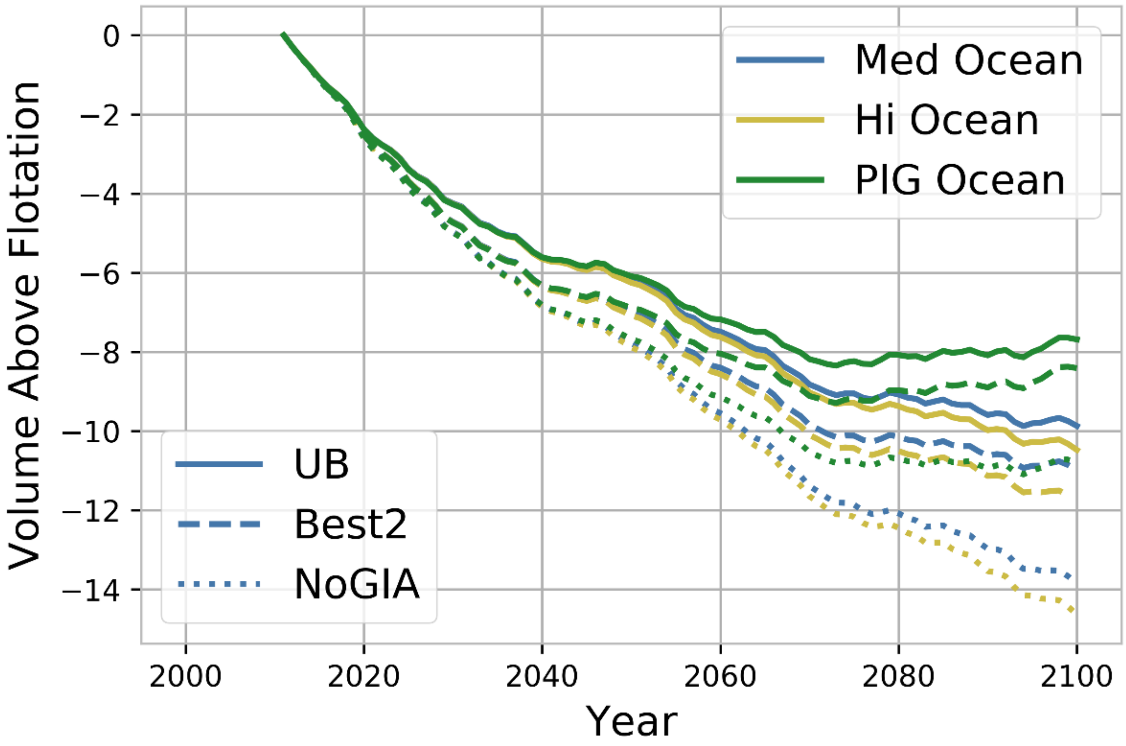
Kachuck, et al. (GRL, 2020)

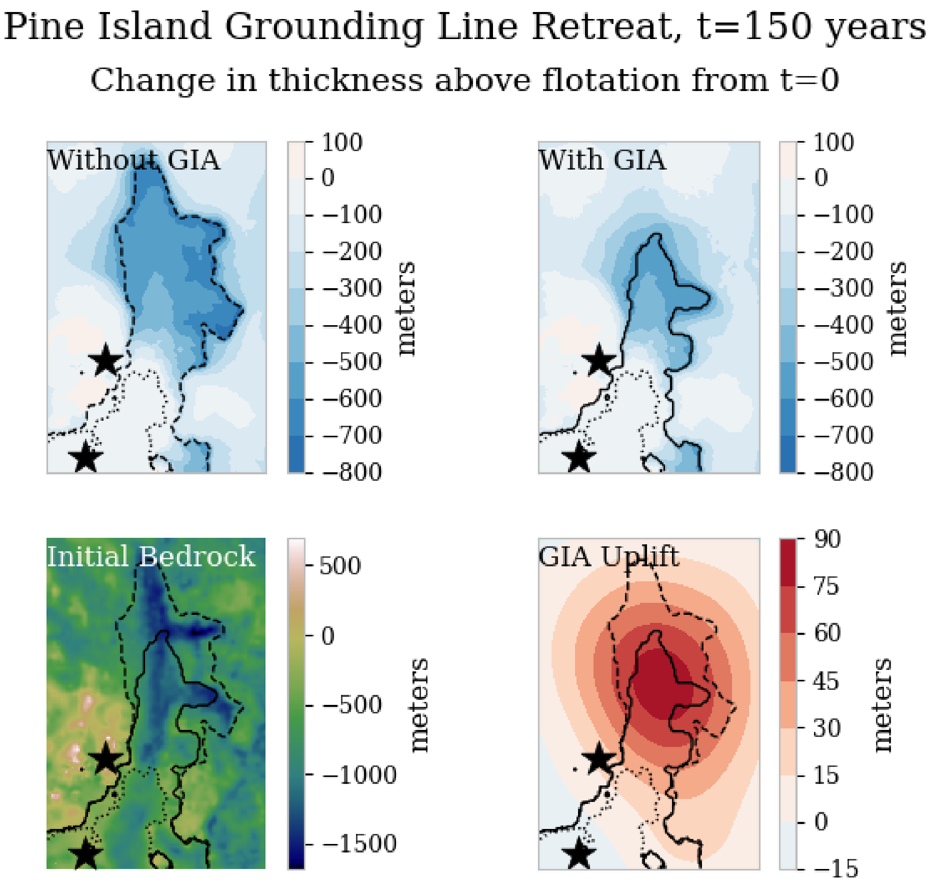
GIA Slows retreat of Pine Island



Kachuck, et al. (GRL, 2020)

GIA Slows retreat of Pine Island & Thwaites





GPS stations ☆ miss most of possible uplift!

Kachuck, et al. (GRL, 2020)

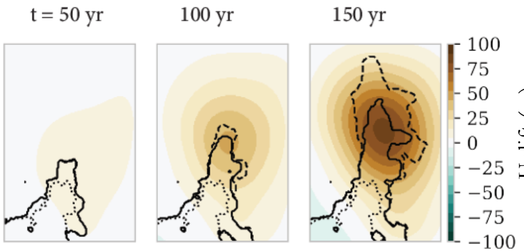
Rapid Viscoelastic Deformation Slows Marine Ice Sheet Instability at Pine Island Glacier, West Antarctica

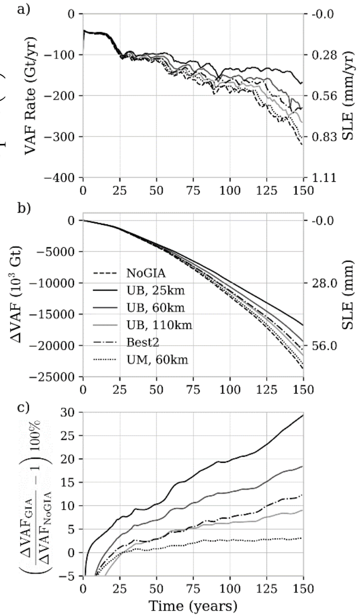
Scientific Achievement

We examine the feedback between ice sheet dynamics and solid-earth viscoelastic response and its impact on grounding-line (GL) stability and sea level rise (SLR).

Significance and Impact

- **Solid-earth feedbacks, from viscoelastic glacial isostatic adjustment (GIA) in response to unloading from ice sheet thinning, have the potential to mitigate GL retreat and mass loss due to marine ice sheet instability.**
- **Full dynamic coupling of a viscoelastic solid-earth deformation model to the DOE-supported BISICLES ice sheet model in a new GIANT-BISICLES model.**
- **Bedrock uplift due to the viscoelastic response of a low-viscosity mantle can rapidly reduce ocean depth near the grounding line and stabilize the marine ice sheet instability over decades to centuries.**
- **Viscoelastic uplift on timescales similar to grounding line migration can be a leading term in determining rates of SLR due to the marine ice sheet instability.**





***Top:** Viscoelastic uplift of the grounding line (GL) slows retreat. The regional uplift at three times: t=50, 100, and 150 years for model with solid-earth response, also showing the initial GL (dotted) and predicted GL with uplift (GIA -solid) and without uplift (NoGIA - dashed) contours shown. The maximum uplift is predicted just in front of the grounding line.*
***Right:** Results from coupling Pine Island Glacier flow to*
GIA-related deformation over 150 years for different rheologies. a) Volume above flotation (VAF) loss rate in Gigatons of ice and millimeters of equivalent sea level rise (SLE). b) Change in total VAF (ΔVAF) relative to t=0. c) Percentage difference ΔVAF (from b) between models with GIA-related deformation relative to without.

Research Details

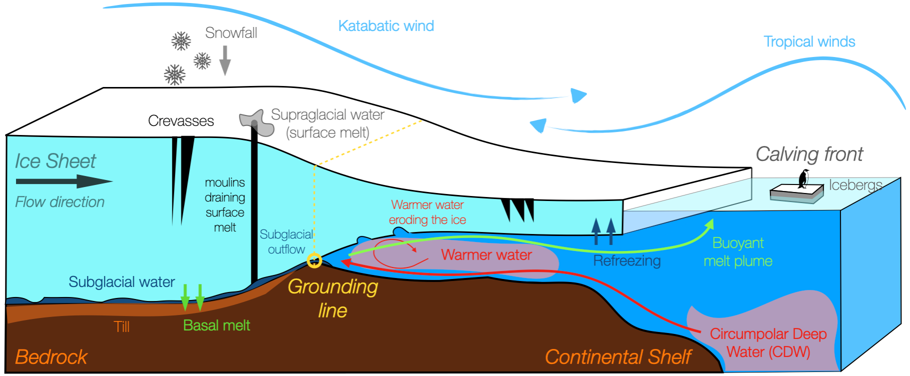
- **Coupled solid-earth/ice sheet model used to simulate West Antarctica's Pine Island Glacier for a range of potential mantle viscosities.**
- **Uses DOE SciDAC-supported BISICLES adaptive mesh refinement (AMR) ice sheet model which resolves flow down to 500m resolution, essential for accurately capturing realistic grounding line dynamics.**

Kachuck, Martin, Bassis, and Price (2020). Geophysical Research Letters, **DOI 10.1029/2019GL086446**. Contact: Samuel Kachuck (skachuck@umich.edu)

Subglacial Hydrology

- Water under the ice can lubricate the bed of the ice, changing the basal friction
- Greenland summer melt
- Antarctic Subglacial lakes
- **Very dynamic** – transitions between inefficient “distributed” system and efficient “channelized” system

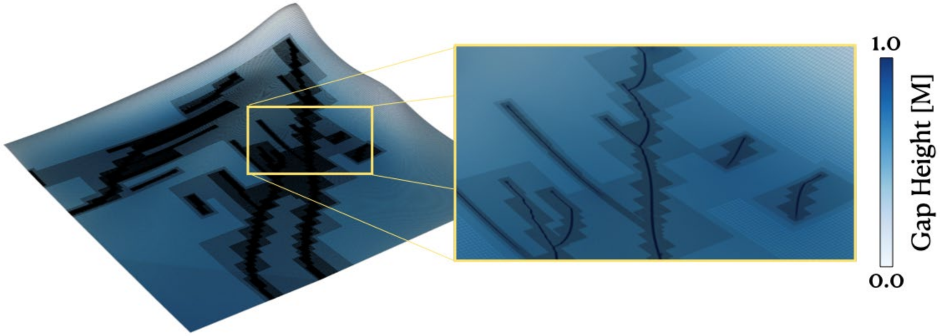
- Want to be able to model this!



SUHMO

- **Su**bglacial Hydrology Model
- AMR model
  - Adapts mesh as channels evolve

- **Status:**
  - Testing
  - Writing paper
  - **Couple to BISICLES!**



Global Earth System Models

- **The Energy Exascale Earth System Model (E3SM)**
  - DOE’s high-resolution global earth system model
  - Currently in development
  - Effort underway at LBL to couple BISICLES as an ice sheet model component
- **UK Earth System Model (UKESM)**
  - First fully-coupled Antarctic response
  - Coupling is hard!

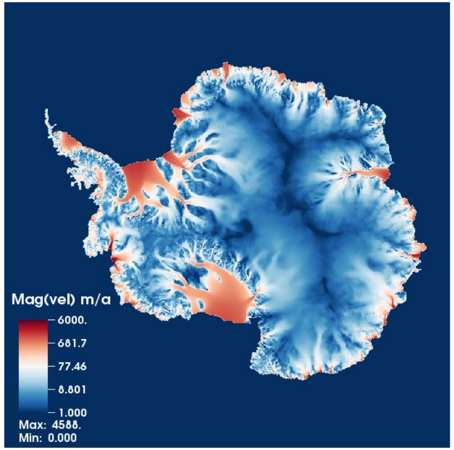
So what can we do with an AMR ice sheet model?

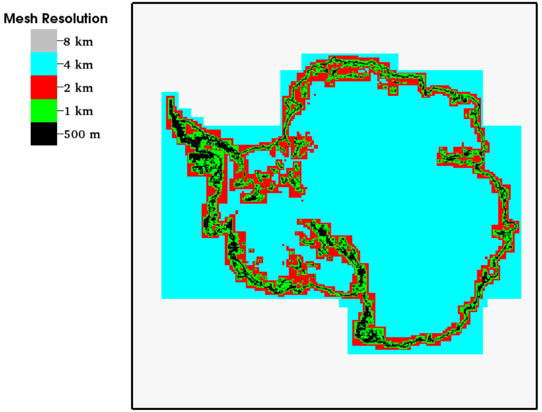
- **Couple with ocean & earth system models…**
- **Examine resolution requirements and convergence of full-scale problems…**

“Adaptive mesh refinement versus subgrid friction interpolation in simulations of Antarctic Ice Dynamics”, Cornford, Martin, Lee, Payne, Ng, *Annals of Glaciology*, 57 (73), 2016

Initial Condition for Antarctic Simulations

- Full-continent Bedmap2 (2013) geometry
- Temperature field from Pattyn (2010)
- Initialize basal friction to match Rignot (2011) velocities
- SMB: Arthern et al (2006)
- AMR meshes: 8 km base mesh, adaptively refine to Δx_f





Experiment – 1000-year Antarctic simulations

- **Range of finest resolution from 8 km (no refinement) to 500m (4 levels of factor-2 refinement)**
- **Subgrid basal friction parameterization (e.g. Seroussi et al)**
  - Experience shows that it buys us about a factor of 2x
- **At initial time, subject ice shelves to extreme (outlandish) depth-dependent melting:**
  - No melt for h < 100m
  - Range up to 400m/a where h > 800m.
  - **No melt applied in partially-grounded cells**
- **For each resolution, evolve for 1000 years**